OPTIMIZATION OF MULTILAYER FUSING HEAT-INSULATING COATING

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UDC 536.42:519.3

The article suggests an algorithm for minimizing the weight of multilayer fusing HIC with constraints concerning the surface temperatures of the layers.

When the heat insulation of various structures against the effect of a hot environment is worked out, the problem of determining the dimensions of the layers of multilayer heat insulating coatings (HIC) with minimal weight arises. As a rule, additionally, constraints as to the change of temperature at specified points of the HIC are being specified. In various kinds of heat insulation the outer layer of the HIC may be destroyed after initial heating, and the weight of the coating gradually decreases. This is made use of in designing HIC of minimal weight for one-time use. To select a fusing HIC that is optimal in weight, it is necessary to solve the corresponding variational problem.

Second boundary conditions are specified, at the initial instant the HIC is uniformly heated throughout, thermal contact between the layers is ideal. Then the temperature of the layers can be determined from the solution of the following boundary-value problem:

$$\frac{\partial T_i}{\partial t} = \frac{a_i}{h_i^2}\,\frac{\partial^2 T_i}{\partial x^2},\quad 0<t\leqslant t_\varphi,\quad x\in[0,\ 1],\quad i=1,\ 2,\ \ldots,\ n, \tag{1}$$

$$T_i(x,\ 0)=0,\quad i=1,\ 2,\ \ldots,\ n, \tag{2}$$

$$\frac{\lambda_1}{h_1}\,\frac{\partial T_1}{\partial x}(0,\ t)=q_0(t),\quad 0\leqslant t\leqslant t_\varphi, \tag{3}$$

$$\frac{\lambda_n}{h_n}\,\frac{\partial T_n}{\partial x}(1,\ t)=q_n(t),\quad 0\leqslant t\leqslant t_\varphi, \tag{4}$$

$$T_i(1,\ t)=T_{i+1}(0,\ t),\quad 0\leqslant t\leqslant t_\varphi,\quad i=1,\ 2,\ \ldots,\ n-1, \tag{5}$$

$$\frac{\lambda_i}{h_i}\,\frac{\partial T_i}{\partial x}(1,\ t)=\frac{\lambda_{i+1}}{h_{i+1}}\,\frac{\partial T_{i+1}}{\partial x}(0,t),\quad 0\leqslant t\leqslant t_\varphi,\ i=1,\ 2,\ \ldots,\ n-1. \tag{6}$$

At the instant t_ϕ the temperature of the outer layer of the HIC attains the melting point T_ϕ of the material of the n-th layer. It is assumed that the molten mass is removed without energy expenditure. The mathematical formulation of the problem for the fusing regime has the form

$$\frac{\partial T_i}{\partial t} = \frac{a_i}{h_i^2}\,\frac{\partial^2 T_i}{\partial x^2},\quad t_\varphi<t\leqslant t_k,\quad x\in[0,\ 1],\quad i=1,\ 2,\ \ldots,\ n-1, \tag{7}$$

$$\frac{\partial T_n}{\partial t} = \frac{x h_n'(t)}{h_n(t)}\,\frac{\partial T_n}{\partial x}+\frac{a_n}{h_n^2}\,\frac{\partial^2 T_n}{\partial x^2},\quad t_\varphi<t\leqslant t_k,\ x\in[0,1], \tag{8}$$

$$T_i(x,\ t_\varphi)=\tilde{T}_i(x,\ t_\varphi),\quad x\in[0,\ 1],\ i=1,\ 2,\ \ldots,\ n, \tag{9}$$

$$\frac{\lambda_1}{h_1}\,\frac{\partial T_1}{\partial x}(0,\ t)=q_0(t),\quad t_\varphi\leqslant t\leqslant t_k, \tag{10}$$

$$T_n(1,\ t)=T_\varphi,\quad t_\varphi\leqslant t\leqslant t_k, \tag{11}$$

$$T_i(1,\ t)=T_{i+1}(0,\ t),\quad t_\varphi\leqslant t\leqslant t_k,\quad i=1,\ 2,\ \ldots,\ n-1, \tag{12}$$

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 240-245, February, 1986. Original article submitted January 2, 1985.

0022-0841/86/5002-0174$12.50

$$\frac{\lambda_i}{h_i}\frac{\partial T_i}{\partial x}(1, t) = \frac{\lambda_{i+1}}{h_{i+1}}\frac{\partial T_{i+1}}{\partial x}(0, t), \quad t_\varphi \leqslant t \leqslant t_k, \quad i = 1, 2, \ldots, n-1, \tag{13}$$

$$-L_n\rho_n\frac{dh_n}{dt} = q_n(t) - \frac{\lambda_n}{h_n}\frac{\partial T_n}{\partial x}(1, t), \quad t_\varphi < t \leqslant t_k, \tag{14}$$

$$h_n(t_\varphi) = \tilde{h}_n, \tag{15}$$

where $\tilde{T}_i(x, t_\varphi)$, i = 1, 2, ..., n, $\tilde{h}_n$ are, respectively, the temperature of the HIC obtained from the solution of the problem (1)-(6) and the thickness of the n-th layer before fusion.

We have to determine the thicknesses of the layers $h_i^* \geq 0$, i = 1, 2, ..., n, from the solution of the problem of minimizing the weight of the HIC we have

$$M = \sum_{i=1}^{n} \rho_i h_i \tag{16}$$

with constraints for temperature

$$T_i(0, t) \leqslant T^i_{\text{per}}, \quad i = 1, 2, \ldots, n, \tag{17}$$

on the solutions of the boundary-value problems (1)-(6), (7)-(15).

For evaluating how the constraints (17) are fulfilled, we introduce the penalty function

$$\Phi = \frac{1}{2}\sum_{i=1}^{n} g_i^2, \tag{18}$$

where $g_i = \int_0^{t_k} [T^i_{\text{per}} - T_i(0, t)]_- dt; \quad f_- = \min(0, f).$

Then the initial problem reduces to the problem of determining the minimum of function (16) with the condition that $\Phi = 0$. To obtain a solution, an algorithm was used that was based on minimizing the penalty function with a selected value of function (16) with the subsequent selection of the weight in dependence on the minimum of the function Φ.

Step 0. Select the initial approximation $\bar{h}^0 = \{h_i^0, i = 1, 2, \ldots, n\}$, the parameter $\varepsilon > 0$, put k = 0.

Step 1. If $\Phi(\bar{h}^k) > 0$, go to step 2, if $\Phi(\bar{h}^k) = 0$, go to step 3.

Step 2. Calculate $M^k = M(\bar{h}^k)$. Determine the minimum $\bar{h}^{**}$ of the function Φ in the set $\{h: M(\bar{h}) = M^k\}$. If $\Phi(\bar{h}^{**}) > 0$, calculate $\bar{h}^{k+1} = \bar{h}^{**} - \varepsilon\nabla M_1$ and go to step 1. If $\Phi(\bar{h}^{**}) = 0$, calculate $\bar{h}^{k+1} = \bar{h}^{**} - \varepsilon\nabla M_1$ and go to step 1.

Step 3. Calculate $\bar{h}^{k+1} = \bar{h}^{(k)} - \varepsilon\nabla M_1$ and go to step 1.

Calculation according to the algorithm is terminated on condition that from k = N onward, $M^{2k} = M^{2k+2}$, $M^{2k+1} = M^{2k+3}$; here, the accuracy of determining the minimum weight of the HIC with the specified constraints depends on the parameter ε.

The minimum of the function Φ at the step 2 can be found by one of the gradient methods, with the projection of $\nabla\Phi_{1,M}$ of the gradient $\nabla\Phi$ on the plane with the normal ∇M used as gradient of the minimizing function:

$$\nabla\Phi_{1,M} = [[\nabla M_1, \nabla\Phi_1], \nabla M_1], \tag{19}$$

where $\nabla M_1 = \nabla M/|\nabla M|; \quad \nabla\Phi_1 = \nabla\Phi/|\nabla\Phi|.$

To determine the gradient $\nabla\Phi$, we write, in analogy to [1-5], the boundary-value problems conjugate to problems (1)-(6), (7)-(15). Here it must be taken into account that as a result of varying the thicknesses h_i, i = 1, 2, ..., n, we vary, in addition to the temperature, also the time of fusion and the regularity of the motion of the phase transition:

$$-\frac{\partial\psi_i}{\partial t} = \frac{a_i}{h_i^2}\frac{\partial\psi_i}{\partial x^2}, \quad 0 \leqslant t < t_\varphi, \quad x \in [0, 1], \quad i = 1, 2, \ldots, n, \tag{20}$$

$$\psi_i(x, t_\varphi) = \tilde{\psi}_i(x, t_\varphi), \quad x \in [0, 1], \quad i = 1, 2, \ldots, n, \tag{21}$$

$$a_1 \frac{\partial \psi_1}{\partial x}(0, t) = -g_1, \quad 0 \leqslant t \leqslant t_\varphi, \tag{22}$$

$$a_n \frac{\partial \psi_n}{\partial x}(1, t) = 0, \quad 0 \leqslant t \leqslant t_\varphi, \tag{23}$$

$$\frac{a_i h_i}{\lambda_i} \psi_i(1, t) = \frac{a_{i+1} h_{i+1}}{\lambda_{i+1}} \psi_{i+1}(0, t), \ i = 1, 2, \ldots, n-1, \tag{24}$$

$$a_i \frac{\partial \psi_i}{\partial x}(1, t) = a_{i+1} \frac{\partial \psi_{i+1}}{\partial x}(0, t) + g_{i+1}, \ i = 1, 2, \ldots, n-1, \tag{25}$$

$$-\frac{\partial \psi_i}{\partial t} = \frac{a_i}{h_i^2} \frac{\partial^2 \psi_i}{\partial x^2}, \quad t_\varphi \leqslant t \leqslant t_k, \ x \in [0, 1], \ i = 1, 2, \ldots, n-1, \tag{26}$$

$$-\frac{\partial}{\partial t}(h_n^2(t) \psi_n) + h_n h_n' \frac{\partial}{\partial x}(x \psi_n) = a_n \frac{\partial^2 \psi_n}{\partial x^2}, \ t_\varphi \leqslant t \leqslant t_k, \ x \in [0, 1], \tag{27}$$

$$\psi_i(x, t_k) = 0, \ x \in [0, 1], \ i = 1, 2, \ldots, n, \tag{28}$$

$$a_1 \frac{\partial \psi_1}{\partial x}(0, t) = -g_1, \ t_\varphi \leqslant t \leqslant t_k, \tag{29}$$

$$a_n \psi_n(1, t) = -\lambda_n \mu(t), \quad t_\varphi \leqslant t \leqslant t_k, \tag{30}$$

$$\frac{a_i h_i}{\lambda_i} \psi_i(1, t) = \frac{a_{i+1} h_{i+1}}{\lambda_{i+1}} \psi_{i+1}(0, t), \ t_\varphi \leqslant t \leqslant t_k, \ i = 1, 2, \ldots, n-1, \tag{31}$$

$$a_i \frac{\partial \psi_i}{\partial x}(1, t) = a_{i+1} \frac{\partial \psi_{i+1}}{\partial x}(0, t) + g_{i+1}, \ t_\varphi \leqslant t \leqslant t_k, \ i = 1, 2, \ldots, n-1, \tag{32}$$

$$L_n \rho_n h_n \frac{d\mu}{dt} = \mu(t) q_n(t) + \frac{a_{n-1}}{h_n} \psi_{n-1}(1,t) \frac{\partial T_{n-1}}{\partial x}(1, t) + \int_0^1 \left(2h_n \frac{\partial T_n}{\partial t} \psi_n + x h_n \frac{\partial}{\partial t}\left(\psi_n \frac{\partial T_n}{\partial x}\right)\right) dx, \tag{33}$$

$$\mu(t_k) = 0. \tag{34}$$

Then the gradient $\nabla\Phi = \{\partial\Phi/\partial h_i, \ i = 1, 2, \ldots, n\}$ is calculated from the solutions of the boundary-value problems (1)-(6), (7)-(15) by the formulas

$$\frac{\partial \Phi}{\partial h_1} = \int_0^{t_k} \left[\int_0^1 2h_1 \frac{\partial T_1}{\partial t} \psi_1 dx + \frac{a_1}{\lambda_1} q_0(t) \psi_1(0, t) - \frac{a_2}{h_1} \psi_2(0, t) \frac{\partial T_2}{\partial x}(0, t) \right] dt, \tag{35}$$

$$\frac{\partial \Phi}{\partial h_i} = \int_0^{t_k} \left[\int_0^1 2h_i \frac{\partial T_i}{\partial t} \psi_i dx + \frac{a_{i-1}}{h_i} \psi_{i-1}(1, t) \frac{\partial T_{i-1}}{\partial x}(1, t) - \frac{a_{i+1}}{h_i} \psi_{i+1}(0, t) \frac{\partial T_{i+1}}{\partial x}(0, t) \right] dt, \quad i = 2, 3, \ldots, n-1, \tag{36}$$

$$\frac{\partial \Phi}{\partial h_n} = \int_0^{t_\varphi} \left[\int_0^1 2h_n \frac{\partial T_n}{\partial t} \psi_n dx + \frac{a_{n-1}}{h_n} \psi_{n-1}(1, t) \frac{\partial T_{n-1}}{\partial x}(1, t) - \frac{a_n}{\lambda_n} q_n(t) \psi_n(1, t) \right] dt + \int_0^1 h_n(t_\varphi) x \psi_n(x, t_\varphi) \frac{\partial T_n}{\partial x}(x, t_\varphi) dx - L_n \rho_n h_n(t_\varphi) \mu(t_\varphi). \tag{37}$$

TABLE 1. Results of the Optmization of Weight

T_φ	h_1	h_2	$h_2(t_k)$	$M\|_{t=0}$	$M\|_{t=t_k}$
1,17	0,75586	0,34162	0,32672	0,71955	0,70465
No fusion	0,71156	0,387	0,387	0,74278	0,74278

The algorithm of minimization using the above formulas was realized in the form of a computer program with which a number of methodological examples were calculated. The boundary-value problems were solved by the combined numerical and analytical method [6, 7] of seeking the solution in the form of a power series with respect to the space coordinate. This made it possible to take the integrals with respect to x in formulas (33), (35)-(37) analytically, which saved computer time and increased the accuracy of determining the gradient. To find the minimum of the function Φ, the method of steepest descent was used. This does not exclude the possibility of using more effective gradient methods of determining the extremum at this step.

As an example we solve the problem for a two-layer HIC with the following initial data: $\underline{a}_1 = \underline{a}_2 = 1$, $\lambda_1 = \lambda_2 = 1$, $\rho_1 = 0.5$, $\rho_2 = 1$, $L_2 = 2$, $T_\phi = 1.17$, $q_0(t) = 0$, $q_2(t) = 1$, $t_k = 1$, $T^1_{per} = 0.72711$, $T^2_{per} = 0.95856$, $\varepsilon = 0.0025$. For the sake of comparison we solved the same problem but with the provision that the temprature T_ϕ is much higher and that there is no fusion. The results are presented in Table 1.

For a nonfusing two-layer HIC we calculated variants with the initial data of [2] as test problems. The results of the optimization coincide.

It can be seen from the values of the weight in Table 1 that fusing HIC is lighter than nonfusing one, and in addition, its weight decreases in time. When the material of the layers is adequately chosen, this effect may be considerable, and it may be used in designing HIC with minimum weight for a single use.

Our calculations showed that the algorithm for minimizing the linear function with nonlinear convex constraints is highly effective. Whereas the existing algorithms [8-11] for solving the problem with the aid of penalty functions reduce the initial problem to a sequence of problems of unconditional minimization of the sum of the penalty function and of the minimizing function with changing penalty parameters, our algorithm does not require the penalty parameters to be selected. In addition, the accuracy of determining the minimum of the penalty function at each step is different. Higher accuracy in finding this minimum is required in the vicinity of the minimal weight.

NOTATION

T, temperature; a, thermal diffusivity; λ, thermal conductivity; ρ, density; L, specific heat of fusion; T_ϕ, melting point; x, coordinate; t, time; t_ϕ, onset of fusion; t_k, right-hand limit of a time interval; i = 1, 2, ..., n, number of the layer of the HIC; M, weight of the HIC; q(t), specific heat flux; T_{per}, specified maximal permissible temperature; h, thickness of the layer; Φ, penalty function; g, constraint function; ∇Φ, gradient of the penalty function; ψ, μ, conjugate functions.

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EFFECT OF THE FREQUENCY OF THE EXTERNAL MAGNETIC FIELD ON THE BEHAVIOR OF THE ARC IN A TWO-JET PLASMATRON

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UDC 533.9

The results of studies of the frequency characteristics of an electric arc in a two-jet plasmatron in a transverse magnetic field are presented.

Interest in the dynamic characteristics of an electric arc has increased significantly in recent years. Thus in [1] the frequency characteristics of the shunting of the arc in the output electrode of the plasmatron with interelectrode inserts were studied. The frequency characteristics of the radial pulsations of the arc column in a longitudinal gas flow in different characteristic sections of the channel are also presented here. In [2], together with an analysis of the shunting of an arc discharge in a longitudinal channel, the oscillations of the electric arc in a transverse gas flow are described. In the cases studied the pulsational characteristics of the electric arc are determined primarily by the hydrodynamic parameters of the gas flows bathing the arc, for example, the degree of its turbulence [1], and depend on the form and proximity of the walls of the arc channel. The imposition of a magnetic field on the arc has a substantial effect on the behavior of the electric arc in a channel [1, 3]. In addition, diverse effects accompanying the burning of the arc in a limited channel, are superposed on the frequency characteristics of the electric arc itself, and they cannot be distinguished in a pure form.

The behavior of an open arc, stabilized by an accompanying gas flow, in a magnetic field has never been adequately studied, though it is of general scientific and practical interest because of the widespread use of and prospects for two-jet plasmatrons.

It is shown in [4] that the imposition of a transverse alternating magnetic field on the anodic and cathodic sections of an electric arc in a two-jet plasmatron enables the realization of effective control of the motion of its sections and the regulation of the power generated in it. The mathematical dependences describing the change in the angle of inclination of the electric field under the action of a constant or weakly varying magnetic field (the frequency of the external alternating magnetic field is equal to 50 Hz) are also presented there.

L. I. Brezhnev Dnepropetrovsk Metallurgical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 245-249, February, 1986. Original article submitted November 6, 1984.

0022-0841/86/5002-0178$12.50